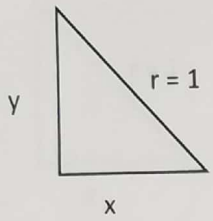
Pythagorean Investigation

1. Given the triangle below, fill in the missing pieces.
   Remember $cos^2\theta = x^2$ and $sin^2\theta = y^2$



If: $x^2 +$ y^2 $= r^2$

And: $x^2 + y^2 =$ 1

Then: $cos^2\theta + sin^2\theta =$ 1

$(\cos\theta)^2 = \cos^2\theta \quad (\sin\theta)^2 = \sin^2\theta$

2. If you divide everything in the equation: $cos^2\theta + sin^2\theta = 1$, by $cos^2\theta$, what would be the result?

$\frac{cos^2\theta}{cos^2\theta} + \frac{sin^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta}$ ⟶ $1 + \tan^2\theta = \sec^2\theta$

3. If you divide everything in the equation: $cos^2\theta + sin^2\theta = 1$, by $sin^2\theta$, what would be the result?

$\frac{cos^2\theta}{sin^2\theta} + \frac{sin^2\theta}{sin^2\theta} = \frac{1}{sin^2\theta}$ ⟶ $\cot^2\theta + 1 = \csc^2\theta$

In conclusion: Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

careful:

$-\cos^2\theta = \sin^2\theta - 1$

$-\sin^2\theta = \cos^2\theta - 1$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$1 = \sec^2\theta - \tan^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$1 = \csc^2\theta - \cot^2\theta$

Section 5.1 Notes Examples

MUST SHOW EVERY STEP on Lined paper Neatly.

9) $\tan x \cos x$

rewrite $\tan x = \frac{\sin x}{\cos x}$

$\frac{\sin x}{\cos x} \cdot \cos x$

$\boxed{\sin x}$

11) skip

13) $\frac{1+\tan^2 x}{\csc^2 x} = \sec^2 x$

Pythagorean identity

$\cdot \frac{\sec^2 x}{\csc^2 x}$

$\cdot \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$

rewrite sine/cosine

$\cdot \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1}$

$\cdot \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$

15) $\cos x - \cos^3 x$

factor GCF

Pythagorean Identity

$\cos x(1 - \cos^2 x)$

$\boxed{\cos x \cdot \sin^2 x}$

24) $\cdot \frac{1+\tan x}{1+\cot x}$

rewrite sine/cosine

$\cdot \frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x} 1 + \frac{\cos x}{\sin x}}$

Common Denominator in N & D

$\cdot \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}}$

Multiply by reciprocal of D

$\cdot \frac{(\cos x + \sin x)}{\cos x} \cdot \frac{\sin x}{(\sin x + \cos x)} = \frac{\sin x}{\cos x} = \boxed{\tan x}$

25) $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$

regroup: $\sec^2 x + \csc^2 x - \tan^2 x - \cot^2 x$

Pythagorean Identities: $\underbrace{\sec^2 x - \tan^2 x}_{=1} + \underbrace{\csc^2 x - \cot^2 x}_{=1}$

$1 + 1$

$\boxed{2}$

HW Sec 5.1 #10-16, 26

- must show all steps for credit.
- must be neat, working vertically on lined paper.

Strategy

- look for Pythagorean Identities
- rewrite in terms of sine/cosine
- Know your Algebra
  - ★ factoring
  - ★ common denominators when adding
  - ★ dividing fractions
  - ★ Complex fractions